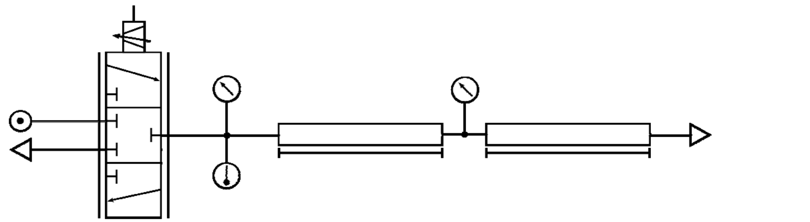
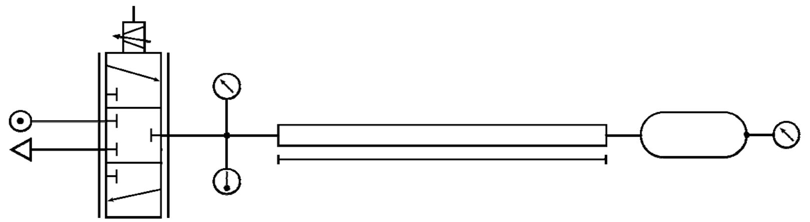
Chair of Automatic Control

Technische Universität München

Modelling of a Long Pneumatic Transmission Line: Models of Successively Decreasing Complexity and their Experimental Validation





Ampère Lab

Lyon, 27th of October 2016

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Overview

Motivation

Purpose of a model

- Analysis
- Feedforward control
- Feedback control
- Optimization

Problems

- Which model is well-suited to describe a pneumatic transmission line?
- „Le simple est toujours faux. Ce qui ne l'est pas est inutilisable.“ Paul Valéry (1937)

$$\rho_t + (\rho v)_z = 0$$
$$(\rho v)_t + (\rho v^2 + p)_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$
$$(\rho e)_t + (v(\rho e + p))_z = \frac{1}{A} 2\pi\alpha r_i (T_0 - T) + f_{\text{comp}} \frac{\rho v^2 |v|}{2D}$$

Simplifications →

$$\rho_t + (\rho v)_z = 0$$
$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} \frac{1}{\rho_0} \rho v$$

Overview

Agenda

Overview

- Motivation
- Agenda

One-dimensional compressible flow

- Conservation laws
- Friction and heat transfer
- Successive model simplifications

Simulation and measurement results

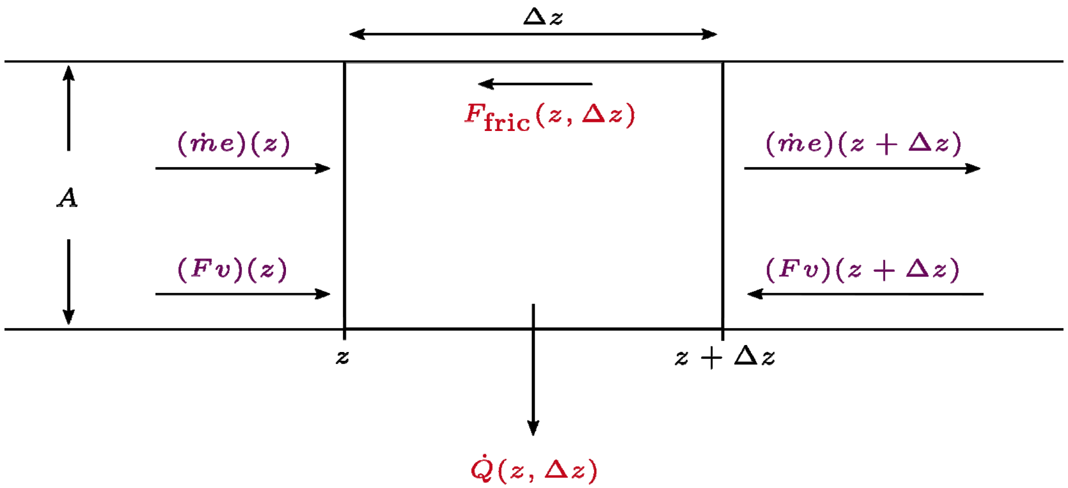
- Scenario 1 – Flow into ambient air
- Scenario 2 – Flow into a terminating volume

Summary and outlook

References

One-dimensional compressible flow

Conservation laws



Euler equations

- Conservation of mass
  $\rho_t + (\rho v)_z = 0$
- Conservation of momentum
  $(\rho v)_t + (\rho v^2 + p)_z = 0$
- Conservation of energy
  $(\rho e)_t + (v(\rho e + p))_z = 0$

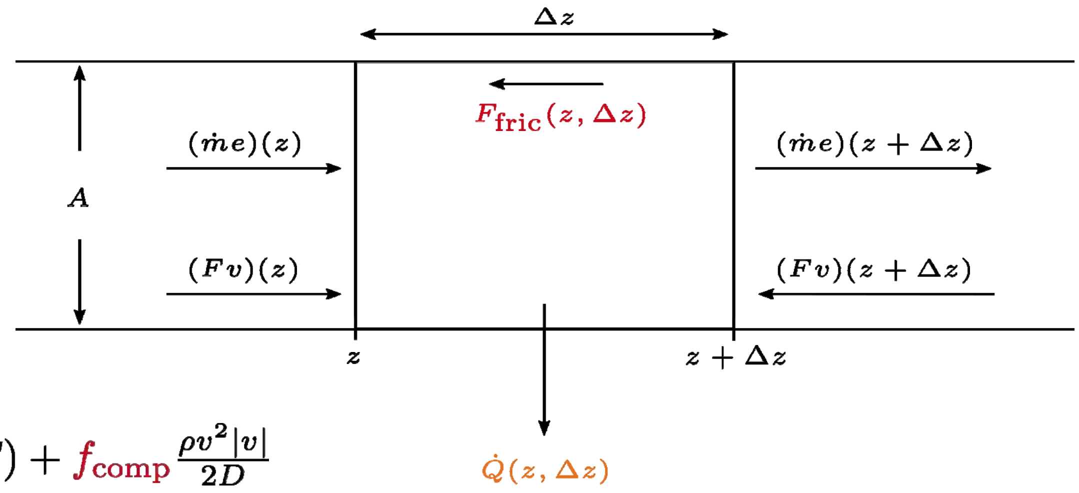
Augmented Euler equations

- Conservation of mass
  $\rho_t + (\rho v)_z = 0$
- Conservation of momentum
  $(\rho v)_t + (\rho v^2 + p)_z = -f_{\text{comp}} \frac{\rho v |v|}{2D}$
- Conservation of energy
  $(\rho e)_t + (v(\rho e + p))_z = \frac{1}{A} 2\pi \alpha r_i (T_0 - T) + f_{\text{comp}} \frac{\rho v^2 |v|}{2D}$

One-dimensional compressible flow

Friction and heat transfer

Augmented Euler equations



- Conservation of mass

  $$\rho_t + (\rho v)_z = 0$$

- Conservation of momentum

  $$(\rho v)_t + (\rho v^2 + p)_z = -f_{\text{comp}} \frac{\rho v |v|}{2D}$$

- Conservation of energy

  $$(\rho e)_t + (v(\rho e + p))_z = \frac{1}{A} 2\pi \alpha r_i (T_0 - T) + f_{\text{comp}} \frac{\rho v^2 |v|}{2D}$$

Empirical correlations

- Compressible friction factor

  $$f_{\text{comp}} = f \left(1 + \frac{\gamma - 1}{2} Ma^2\right)^{-0.47}$$

  - Laminar flow

    $$f = \frac{64}{Re}$$

  - Turbulent flow (Haaland)

    $$\frac{1}{\sqrt{f}} = -1.8 \log \left[\left(\frac{\varepsilon / D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right]$$

- Heat transfer coefficient (Gnielinski)

  $$\alpha = \frac{\lambda_f}{D} \frac{\frac{f_{\text{comp}}}{8} (Re - 1000) Pr}{1 + 12.7 \left(\frac{f_{\text{comp}}}{8} \right)^{\frac{1}{2}} \left(Pr^{\frac{2}{3}} - 1 \right)}$$

One-dimensional compressible flow

Successive model simplifications

Model 1

$$\rho_t + (\rho v)_z = 0$$

$$(\rho v)_t + (\rho v^2 + p)_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

$$(\rho e)_t + (v(\rho e + p))_z = \frac{1}{A} 2\pi \alpha r_i (T_0 - T) + f_{\text{comp}} \frac{\rho v^2 |v|}{2D}$$

Assumptions

- No diffusion
- One-dimensional flow
- No gravity
- Fluid is a polytropic, ideal gas

Model 2

$$\rho_t + (\rho v)_z = 0$$

$$(\rho v)_t + (\rho v^2 + a_{\text{iso}}^2 \rho)_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}, \quad a_{\text{iso}} = \sqrt{\gamma R_s T_0}$$

Additional Assumption

- Flow is isothermal
  - Valid if pressure changes are slow compared to the time needed to reach the thermal equilibrium

One-dimensional compressible flow

Successive model simplifications

Model 2

$$\rho_t + (\rho v)_z = 0$$

$$(\rho v)_t + (\rho v^2 + a_{\text{iso}}^2 \rho)_z = -f_{\text{comp}} \frac{\rho v |v|}{2D}$$

Assumptions

- No diffusion
- One-dimensional flow
- No gravity
- Fluid is a polytropic, ideal gas
- Flow is isothermal

Model 3

$$\rho_t + (\rho v)_z = 0$$

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -f_{\text{comp}} \frac{\rho v |v|}{2D}$$

Additional Assumption

- No convective acceleration
  - Valid for $Ma < 0.3$

One-dimensional compressible flow

Successive model simplifications

Model 3

$$\rho_t + (\rho v)_z = 0$$

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -f_{\text{comp}} \frac{\rho v |v|}{2D}$$

Model 4

$$\rho_t + (\rho v)_z = 0$$

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} v, \quad k_{\text{fric}} \geq 1$$

Assumptions

- No diffusion
- One-dimensional flow
- No gravity
- Fluid is a polytropic, ideal gas
- Flow is isothermal
- No convective acceleration

Additional Assumption

- Laminar flow
  - Valid for small Reynolds numbers
  - Effect of compressibility on the friction factor can be neglected

One-dimensional compressible flow

Successive model simplifications

Model 4

$$\rho_t + (\rho v)_z = 0$$

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} v$$

Model 5

$$\rho_t + (\rho v)_z = 0$$

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} \frac{1}{\rho_0} \rho v$$

Assumptions

- No diffusion
- One-dimensional flow
- No gravity
- Fluid is a polytropic, ideal gas
- Flow is isothermal
- No convective acceleration
- Laminar flow

Additional Assumption

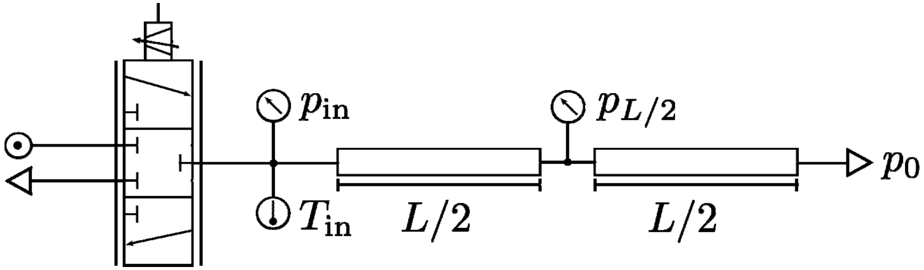
- Density is almost constant
  - Valid for small pressure and temperature changes

Simulation and measurement results

Testing scenarios

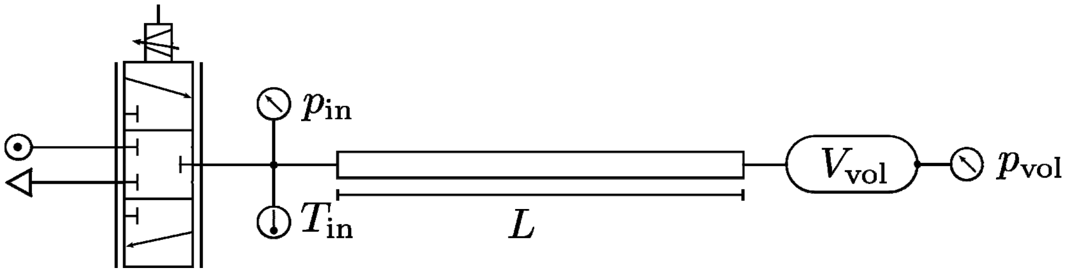
Scenario 1

- Flow in ambient air



Scenario 2

- Flow into a terminating volume



Initial and boundary conditions

- Initial conditions

$$p(z,0) = p_0(z), \quad \rho(z,0) = \rho_0(z), \quad v(z,0) = 0$$

- Left boundary condition

$$p(0,t) = p_{in}(t), \quad \rho(0,t) = \rho_{in}(t)$$

- Right boundary condition scenario 1

$$p(L,t) = p_0$$

- Right boundary condition scenario 2

$$p_{vol}(t) = \frac{m_{vol}(t) R_s T_{vol}(t)}{V_{vol}}$$

$$m_{vol}(t) = \int_0^t \dot{m}(L,\tau)\,\mathrm{d}\tau + m_{vol}(0)$$

$$\left(c_{v,f,vol} m_{vol} T_{vol}\right)_t(t) = (\dot{m}e)(L,t) + (pAv)(L,t) + \dot{Q}_{vol}(t)$$

Simulation and measurement results

Test bench

Mechanical components

- Tube

Diameter	D	$8 \cdot 10^{-3}$ m
Length	L	19,83 m
Roughness	ε	$1{,}5 \cdot 10^{-6}$ m

- Terminating volume

Volume	V_{vol}	$6{,}46 \cdot 10^{-4}$ m^3
Therm. restistance	R_{vol}	$4 \cdot 10^{-3}$ K/W

- Medium air

Ambient temperature	T_0	293,15 K
Ambient pressure	p_0	1,01 bar
Heat capacity ratio	γ	1,4

Electrical components

- Pressure sensors
  - Festo pressure transmitter SPTE
- Ventil
  - Norgren proportional valve VP60
- Computer
  - Intel(R) Core(TM) i7-4770 CPU @ 3.40Ghz
  - Simulink Real-Time

Implementation for simulation

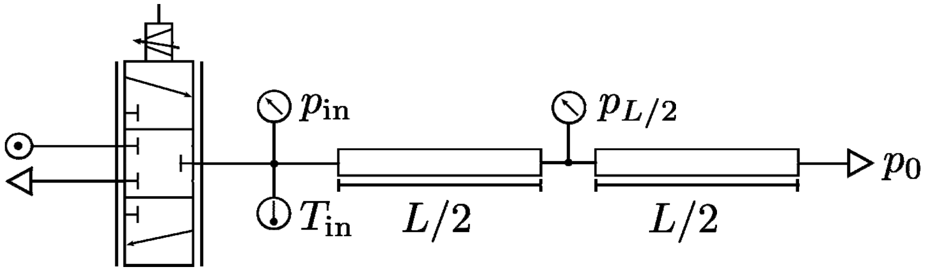
- The PDEs are discretized via the second-order finite difference method of MacCormack

Simulation and measurement results

Testing scenarios

Scenario 1 – Flow into ambient air

- Step input signal
  - Opening the valve in 0,0175 s

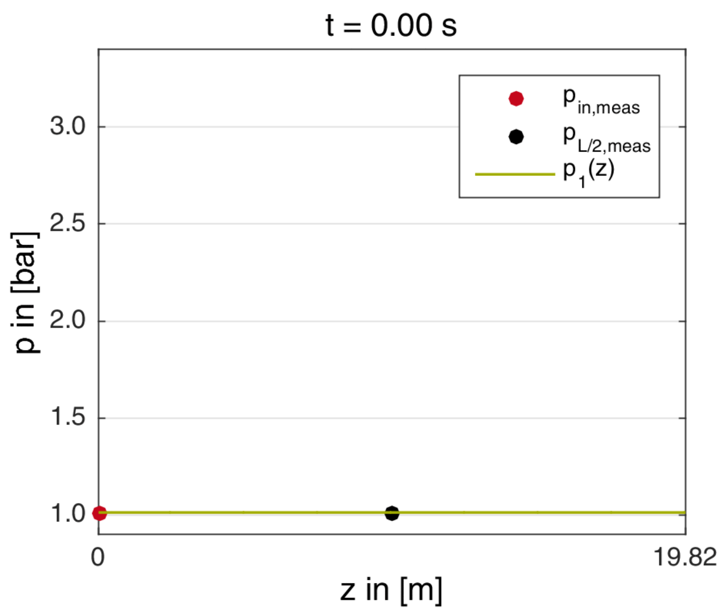


Simulation and measurement results

Testing scenarios

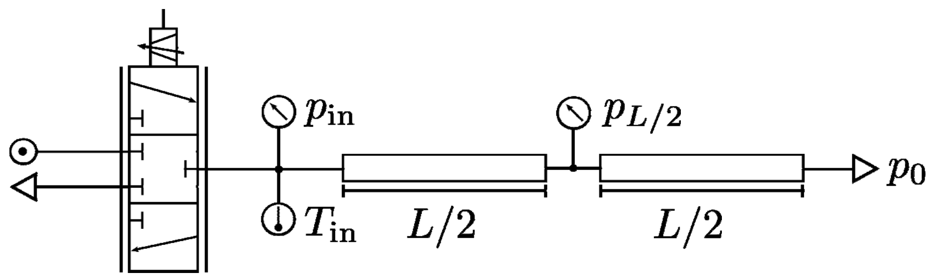
Scenario 1 – Flow into ambient air

- Step input signal
  - Opening the valve in 0,0175 s



- Simulation of model 1
  - v_{max} = 111.06 m/s
  - Ma_{max} = 0.33
  - T_{max} = 317.84 K

Shock wave

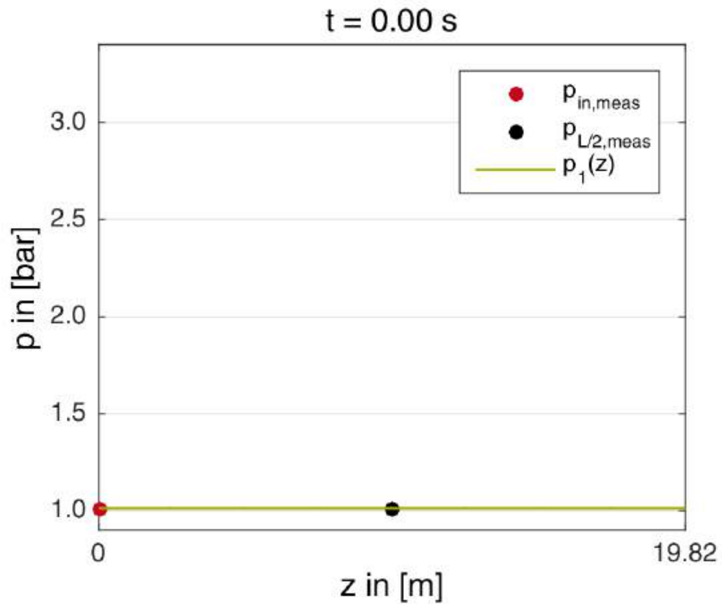


Simulation and measurement results

Testing scenarios

Scenario 1 – Flow into ambient air

- Step input signal
  - Opening the valve in 0.0175 s



- Simulation of model 1
  - v_{max} = 111,06 m/s
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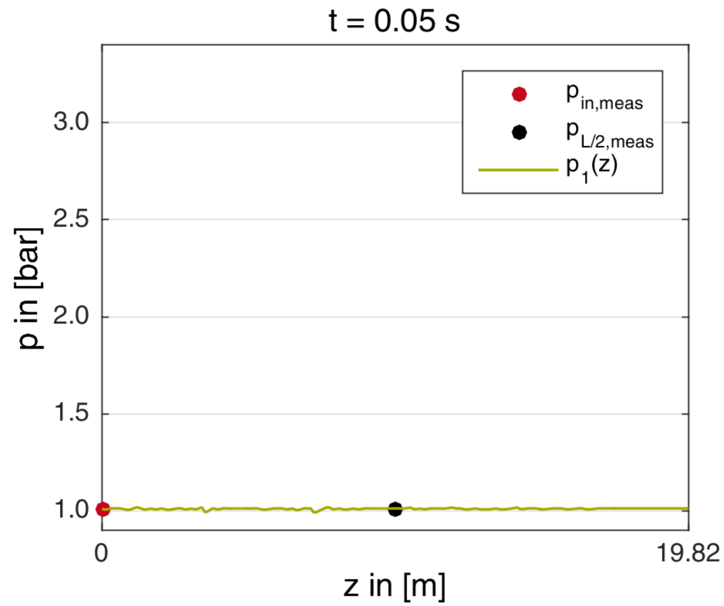
  - T_{max} = 317,84 K

Shock wave

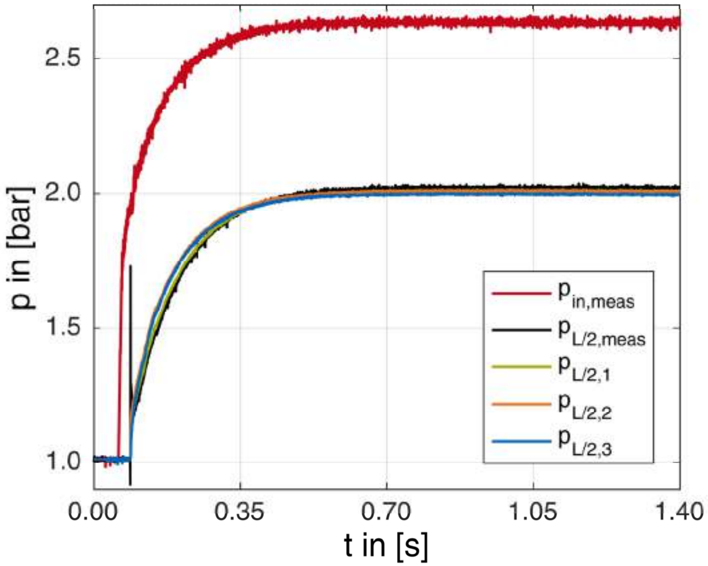


- The shock wave and its reflection are damped due to friction
- Scattering of the shock wave in the sensor causes measurement error
- CFL condition < 1 in the shock wave causes numerical dispersion

Simulation and measurement results

Testing scenarios

Scenario 1 – Flow into ambient air



- Model 1

$$(\rho v)_t + (\rho v^2 + p)_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

$$(\rho e)_t + (v(\rho e + p))_z = \frac{1}{A} 2\pi\alpha r_i (T_0 - T) + f_{\text{comp}} \frac{\rho v^2 |v|}{2D}$$

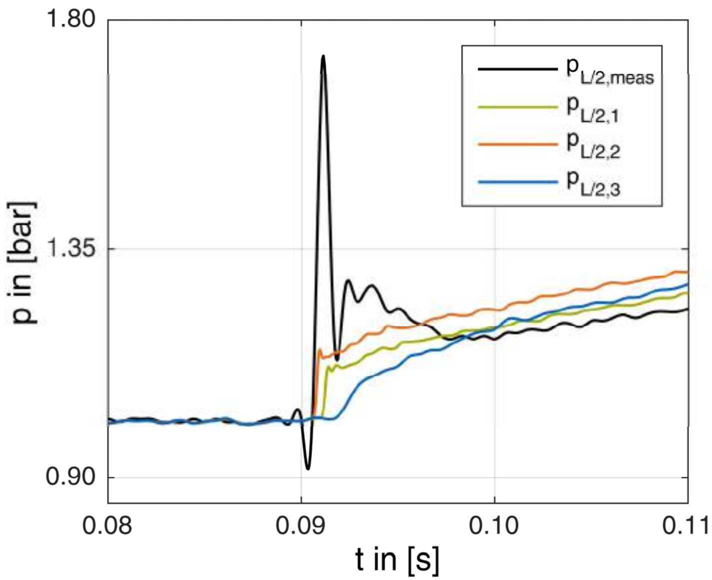
- Model 2

$$(\rho v)_t + (\rho v^2 + a_{\text{iso}}^2 \rho)_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

- Model 3

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

Propagation speed

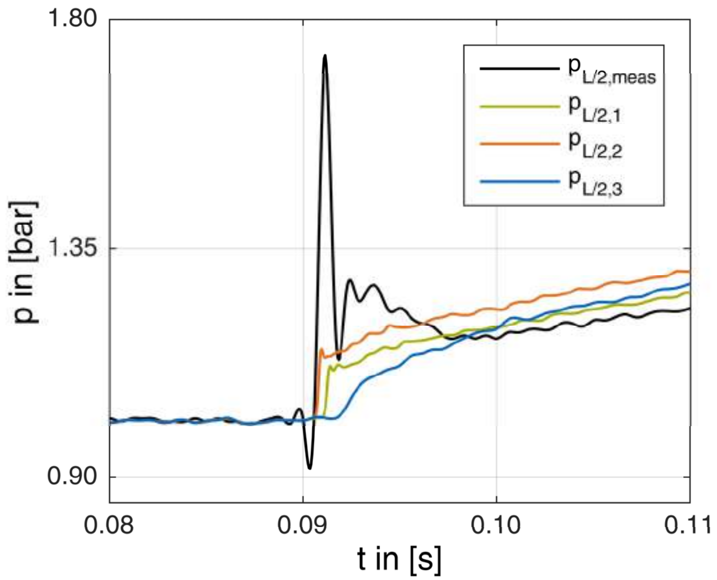


- Model 1: $v + \sqrt{\frac{\gamma p}{\rho}}$
- Model 2: $v + \sqrt{\gamma R_s T_0}$
- Model 3: $\sqrt{\gamma R_s T_0}$

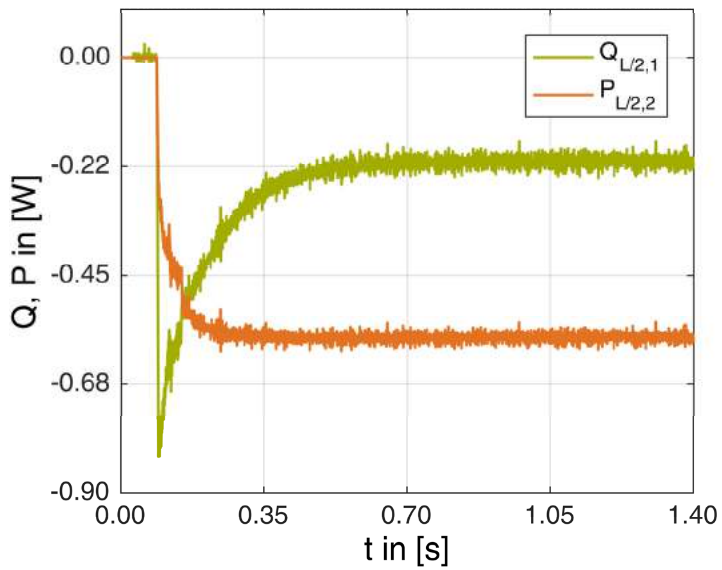
Simulation and measurement results

Testing scenarios

Scenario 1 – Flow into ambient air



Loss of energy



- Model 1

$$(\rho v)_t + (\rho v^2 + p)_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

$$(\rho e)_t + (v(\rho e + p))_z = \frac{1}{A} 2\pi \alpha r_i (T_0 - T) + f_{\text{comp}} \frac{\rho v^2 |v|}{2D}$$

- Model 2

$$(\rho v)_t + (\rho v^2 + a_{\text{iso}}^2 \rho)_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

- Model 3

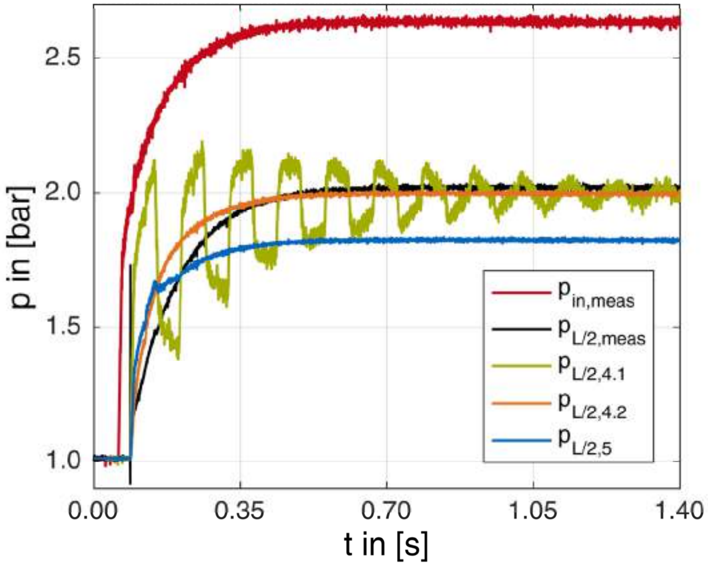
$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

- At $z = \frac{L}{2}$
  - Model 1: $\dot{Q} = \frac{1}{A} 2\pi \alpha r_i (T_0 - T)$
  - Model 2: $P = f_{\text{comp}} \frac{\rho v^2 |v|}{2D}$

Simulation and measurement results

Testing scenarios

Scenario 1 – Flow into ambient air



- Model 4.1

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} v, \quad k_{\text{fric}} = 1$$

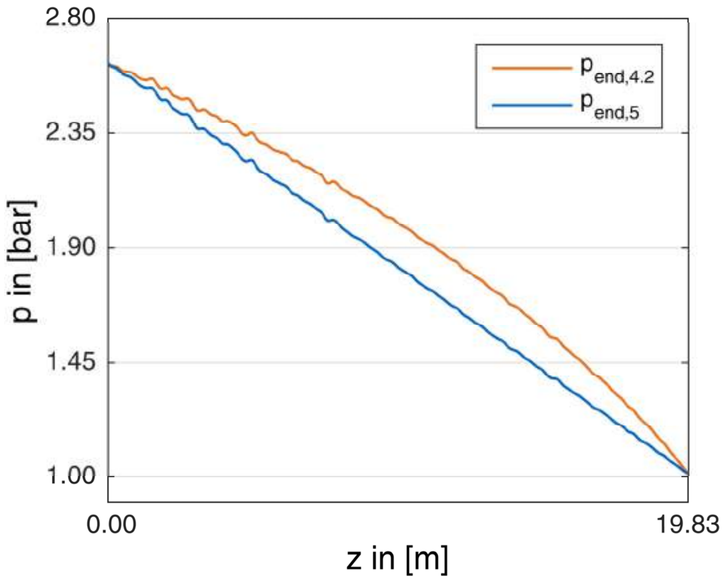
- Model 4.2

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} v, \quad k_{\text{fric}} = 15$$

- Model 5

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} \frac{1}{\rho_0} \rho v, \quad k_{\text{fric}} = 8$$

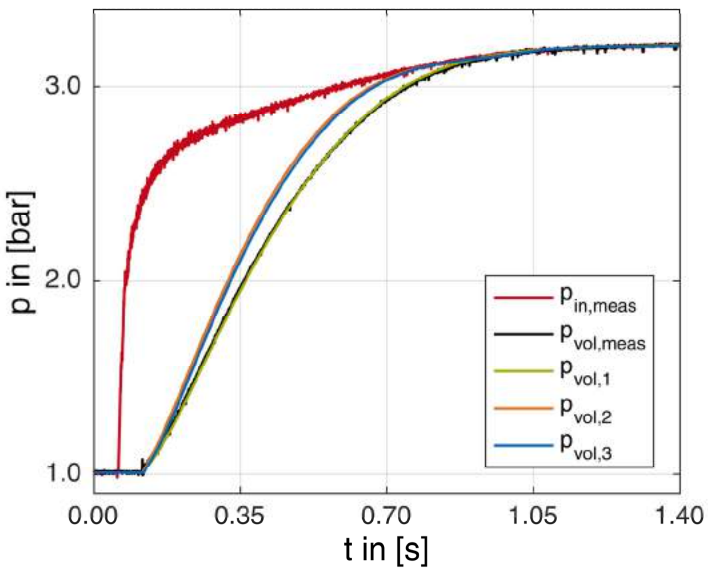
- Choice of k_{fric} for Model 5
  - Model 4: Friction laminar Proportional to v
  - Model 5: Friction linear Proportional zu ρv
- Evident for the steady state solution
  - Pressure profile at t = 1.4 s



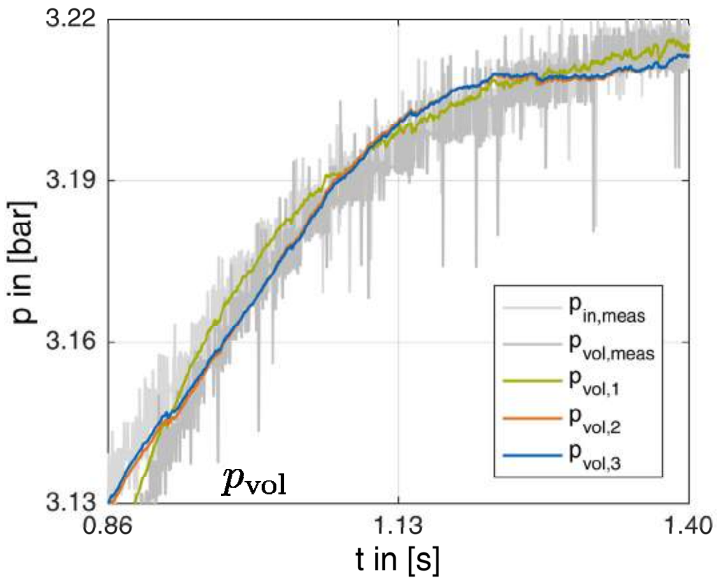
Simulation and measurement results

Testing scenarios

Scenario 2 – Flow into a terminating volume



Pressure in volume



- Model 1

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$$(\rho e)_t + (v(\rho e + p))_z = \frac{1}{A} 2\pi\alpha r_i (T_0 - T) + f_{\text{comp}} \frac{\rho v^2 |v|}{2D}$$

- Model 2

$$(\rho v)_t + (\rho v^2 + a_{\text{iso}}^2 \rho)_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

- Model 3

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -f_{\text{comp}} \frac{\rho v|v|}{2D}$$

- Boundary conditions
  - Model 1:

$$p_{\text{vol}} = \frac{\left(\int_0^t \dot{m}(\tau)\,\mathrm{d}\tau + m_{\text{vol}}\right) R_s T_{\text{vol}}}{V_{\text{vol}}}$$

$$\left(c_{v,f,\text{vol}} m_{\text{vol}} T_{\text{vol}}\right)_t = \dot{m}e + pAv + \dot{Q}_{\text{vol}}$$

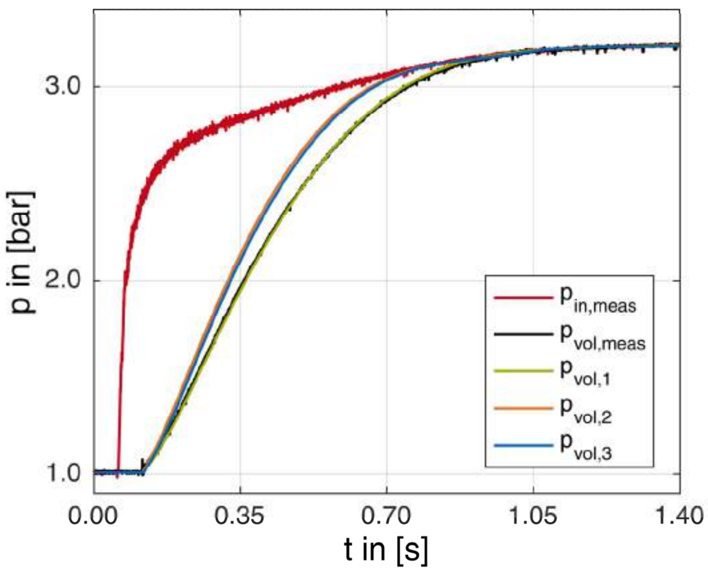
  - Model 2 and 3

$$p_{\text{vol}} = \frac{\left(\int_0^t \dot{m}(\tau)\,\mathrm{d}\tau + m_{\text{vol}}\right) R_s T_0}{V_{\text{vol}}}$$

Simulation and measurement results

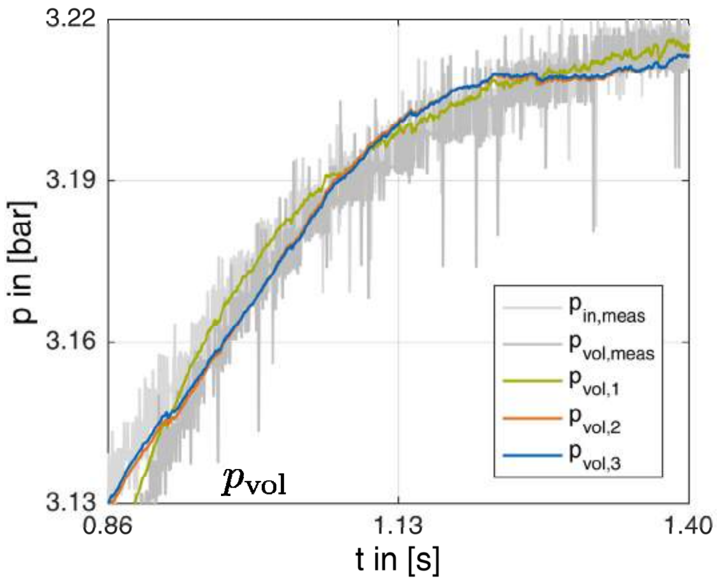
Testing scenarios

Scenario 2 – Flow into a terminating volume



- Deviation from measurement at t = 0.5 s
  - Model 1: 0,45 %
  - Model 2: 8,02 %
- Total energy loss
  - Ratio model 1 to model 2: 1.36

Pressure in volume



- Boundary conditions
  - Model 1:

$$p_{\mathrm{vol}} = \frac{\left(\int_0^t \dot{m}(\tau)\,\mathrm{d}\tau + m_{\mathrm{vol}}\right) R_s T_{\mathrm{vol}}}{V_{\mathrm{vol}}}$$

$$\left(c_{v,f,\mathrm{vol}} m_{\mathrm{vol}} T_{\mathrm{vol}}\right)_t = \dot{m}e + pAv + \dot{Q}_{\mathrm{vol}}$$

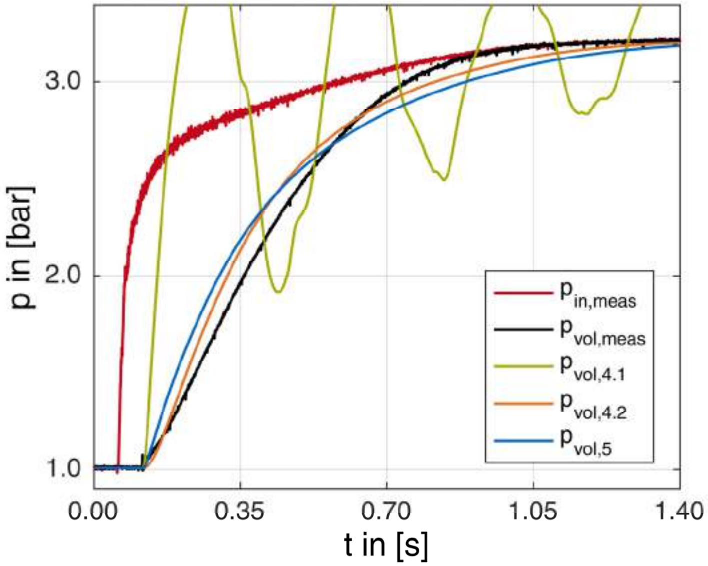
  - Model 2 and 3

$$p_{\mathrm{vol}} = \frac{\left(\int_0^t \dot{m}(\tau)\,\mathrm{d}\tau + m_{\mathrm{vol}}\right) R_s T_0}{V_{\mathrm{vol}}}$$

Simulation and measurement results

Testing scenarios

Scenario 2 – Flow into a terminating volume



- Model 4.1

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} v, \quad k_{\text{fric}} = 1$$

- Model 4.2

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} v, \quad k_{\text{fric}} = 15$$

- Model 5

$$(\rho v)_t + a_{\text{iso}}^2 \rho_z = -k_{\text{fric}} \frac{32\eta_0}{D^2} \frac{1}{\rho_0} \rho v, \quad k_{\text{fric}} = 8$$

Summary and outlook

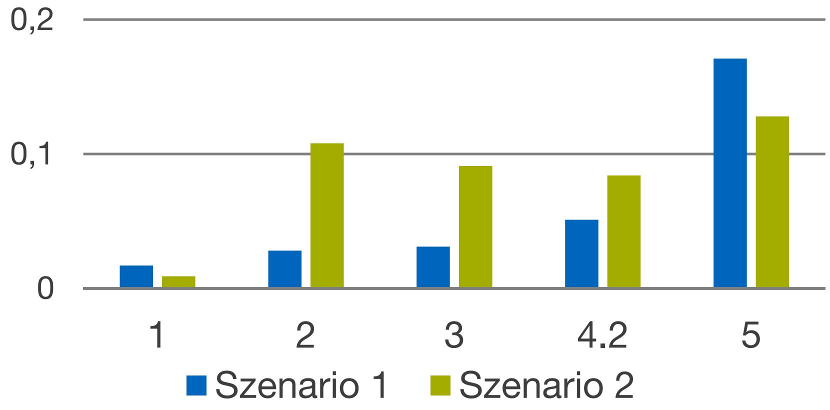
Summary

Assumptions and effects

- Basic assumptions – Model 1
- Isothermal flow – Model 2
  - Two instead of three equations
  - Conservation of energy is violated
- No convective acceleration – Model 3
  - Coefficients of the derivatives are constant (equations are semilinear instead of quasilinear)
  - Wave propagation speed is incorrect
- Laminar flow – Model 4
  - Correlations are not necessary
  - Effect of friction without amplification factor is underestimated
- Constant density – Model 5
  - Linear equations
  - Relatively severe error due to the approximations

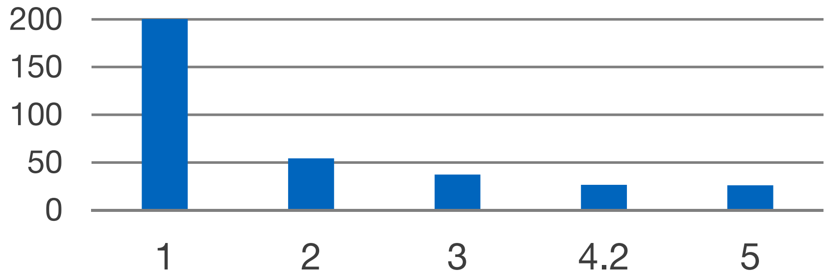
Root mean square

- Difference between simulated and measured pressure (in bar)



Computational time

- Simulation of scenario 1 (in s) with $N_z = 992$ and $N_t = 24\,270 – 30\,514$



Summary and outlook

Outlook

Applications of the models

- Complex models for
  - Simulation
  - Optimization
  - Distributed-parameter feedforward control
  - Lumped-parameter feedback control

- Less complex models
  - If pressure changes are relatively small
  - Suited if the system exhibits a terminating volume at the end of the transmisssion line (e.g. piston)
  - Time-critical applications
  - Distributed-parameter feedback control

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Literature

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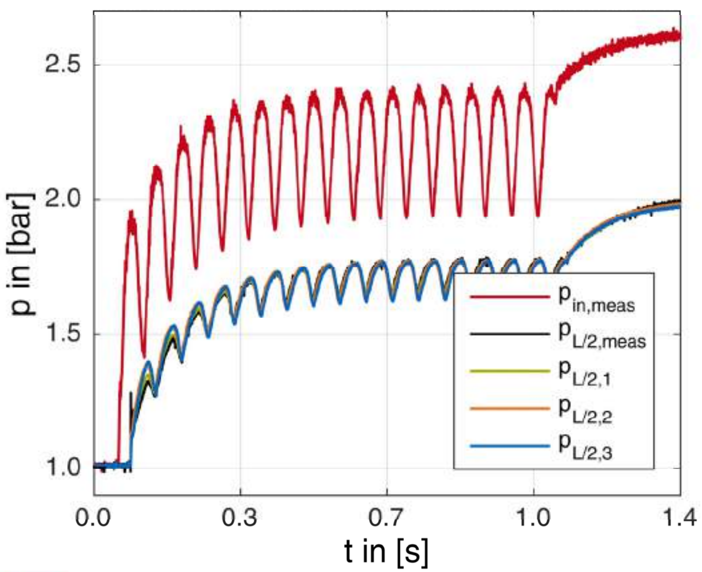
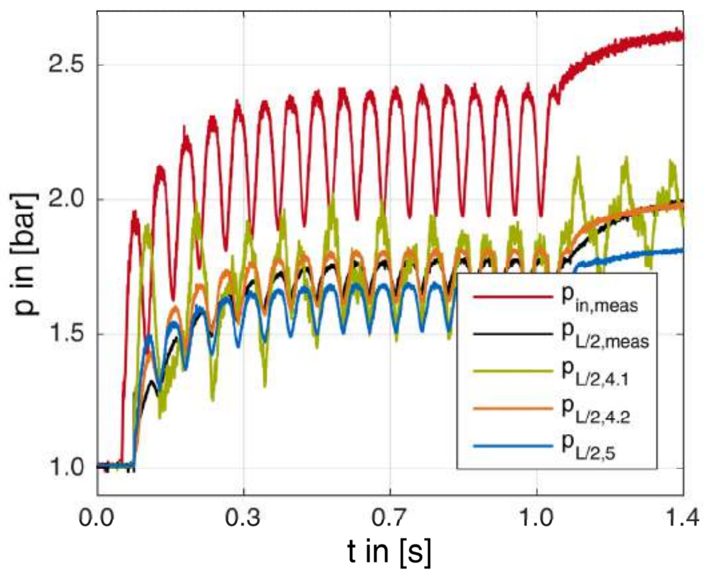
Sinusanregung

Testing scenarios

Scenario 1

- Ecxitation with maximum frequency (~ 16 Hz)



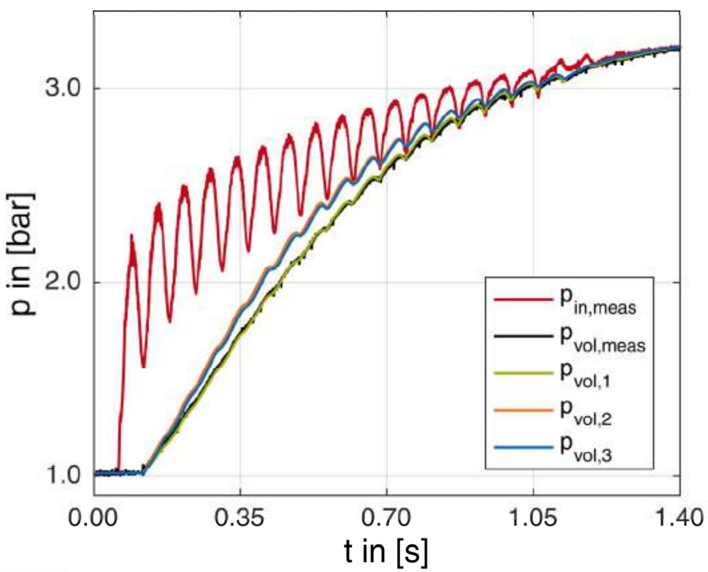


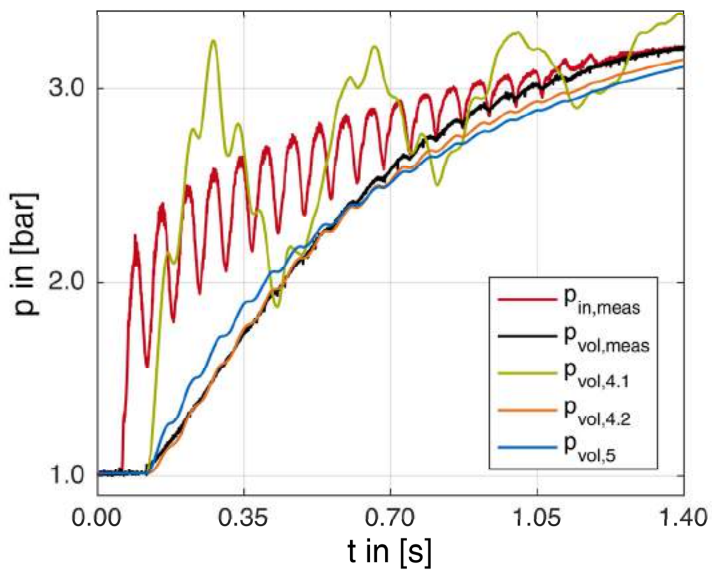
Sinusanregung

Testing scenarios

Scenario 2

- Ecxitation with maximum frequency (~ 16 Hz)





Derivation of Model 3

Linearization of the pressure function

- Formulierung der Eulergleichungen durch Entropieerhaltung

$$\begin{aligned} \rho_t + (\rho v)_z &= 0 \\ (\rho v)_t + (\rho v^2 + p)_z &= 0 \\ S_t + v S_z &= 0 \end{aligned}$$

- Definition der Entropie

$$S = c_v \log\left(\frac{p}{\rho^\gamma}\right) + \kappa$$

- Auflösen nach Druck

$$p(\rho) = \exp\left(\frac{S}{c_v} + \kappa\right)\rho^\gamma$$

- Taylorentwicklung für Funktion des Drucks

$$\begin{aligned} p(\rho) &= p(\rho_0) + \gamma \exp\left(\frac{S}{c_v} + \kappa\right)\rho_0^{\gamma-1}(\rho - \rho_0) + \cdots \\ &= p(\rho_0) + \gamma\frac{p(\rho_0)}{\rho_0}(\rho - \rho_0) + \cdots \end{aligned}$$

- Assumption von kleinen Änderungen

$$p(\rho) = p(\rho_0) + \gamma R_s T_0(\rho - \rho_0)$$

- Isotherme Eulergleichungen

$$\begin{aligned} \rho_t + (\rho v)_z &= 0 \\ (\rho v^2 + a_{\text{iso}}^2 \rho)_z &= 0 \quad \text{with} \quad a_{\text{iso}} = \sqrt{\gamma R_s T_0} \end{aligned}$$