Perturbation Finite Element Method for Skin and Proximity Effects

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Abstract— **Skin and proximity effects are calculated in both active and passive conductors via a subproblem finite element method based on a perturbation technique. A limit problem is first solved by considering perfect conductors via appropriate boundary conditions. Its solution then gives the source for eddy current perturbation subproblems in each conductor with its actual conductivity, each one requiring its own mesh. The proposed method accurately determines the current density distributions and ensuing losses in conductors of any shape in both the frequency and time domains, which overcomes the limitations of the impedance boundary condition technique.**

I. Introduction

A precise consideration of the skin and proximity effects in conductors is usually important for the sources themselves as well as for their surrounding area. This allows an accurate calculation of the ensuing Joule losses in the conductors themselves (either inductors or external conducting pieces, in particular inductively heated pieces), as well as an accurate location of the current density distributions with respect to the influenced regions.

For significant skin and proximity effects, impedance boundary conditions (BCs) [1] defined on the conductor boundaries are an alternative to avoid meshing their interior. Such conditions are nevertheless generally based on analytical solutions of ideal problems and are therefore only valid in practice far from any geometrical discontinuities, as edges and corners. They are also generally limited to frequency domain and linear analyses. In this contribution, a method is developed to overcome the limitations of impedance BCs, allowing conductors of any shape to be considered not only in the frequency domain but also in the time domain. The magnetic vector potential FE magnetodynamic formulation is used.

A limit eddy current FE problem is first solved by considering perfect conductors. This can be done via appropriate conditions on the conductor boundaries, that can serve as well for expressing the circuit relations of active conductors linking their voltages and currents. The solution of the limit problem then gives the source for eddy current FE perturbation subproblems in each conductor with its actual finite conductivity. Each of these problems requires an appropriate volume mesh of the associated conductor and its surrounding region. Such a decoupling allows the solution process to be lightened.

II. The perturbation from perfect to non-perfect conductors

A. The strong formulations

Maxwell equations are to be solved in a bounded domain Ω, with boundary $\partial\Omega$ (possibly at infinity), of the 2-D or 3-D Euclidean space. The eddy current conducting part of Ω is denoted Ω_c and the non-conducting one Ω_c^C, with $\Omega = \Omega_c \cup \Omega_c^C$. Massive conductors belong to Ω_c.

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The equations and relations governing the magnetodynamic problem in Ω are

$$\operatorname{curl} \boldsymbol{h} = \boldsymbol{j}\,,\ \operatorname{curl} \boldsymbol{e} = -\partial_t \boldsymbol{b}\,,\ \operatorname{div} \boldsymbol{b} = 0\,, \qquad \text{(1a-b-c)}$$

$$\boldsymbol{b} = \mu\, \boldsymbol{h}\,,\ \boldsymbol{j} = \sigma\, \boldsymbol{e}\,, \qquad \text{(2a-b)}$$

where $\boldsymbol{h}$ is the magnetic field, $\boldsymbol{b}$ is the magnetic flux density, $\boldsymbol{e}$ is the electric field, $\boldsymbol{j}$ is the electric current density (including source and eddy currents), μ is the magnetic permeability and σ is the electric conductivity. In the following, the subscripts u and p will refer to unperturbed and perturbed quantities, respectively.

Instead of directly solving the eddy current problem with the actual conductivity of some conductors, a so-called unperturbed or limit problem is first defined in Ω by considering some conductors $\Omega_{c,i}$ (i is the conductor index) as being perfect, i.e. of infinite conductivity. This results in a zero skin depth and thus in surface currents. The interior of the conductor regions $\Omega_{c,i}$ can thus be extracted from the studied domain Ω and treated via a BC fixing a zero normal magnetic flux density on their boundaries $\partial\Omega_{c,i}$.

The consideration of the actual conductivity of the concerned conductors, these defining the perturbing region $\Omega_{c,i} \subset \Omega_c$, will further lead to field distortions. The perturbed eddy current problem focuses thus on $\Omega_{c,i}$ and its neighborhood, their union Ω_p being adequately defined and meshed will serve as the studied domain.

The perturbation given by the change of conductivity of the conducting region $\Omega_{c,i}$ alters the distribution of the current density and magnetic field. The fields in these conductors are not surface fields anymore but penetrate them.

Particularizing (1) and (2) for both the unperturbed and perturbed quantities, and subtracting the unperturbed equations from the perturbed ones, a perturbation problem is defined in Ω_p (initially in Ω) [2], [3], [7]. Expressing the resulting equations in terms of the field distortions $\boldsymbol{h} = \boldsymbol{h}_p - \boldsymbol{h}_u$ and $\boldsymbol{e} = \boldsymbol{e}_p - \boldsymbol{e}_u$, one gets

$$\operatorname{curl} \boldsymbol{h} = \boldsymbol{j}\,,\ \operatorname{curl} \boldsymbol{e} = -\partial_t \boldsymbol{b}\,, \qquad \text{(3a-b)}$$

$$\boldsymbol{b} = \mu_p\, \boldsymbol{h} + \boldsymbol{b}_s\,,\ \boldsymbol{j} = \sigma_p\, \boldsymbol{e} + \boldsymbol{j}_s\,, \qquad \text{(4a-b)}$$

where the so-defined volume sources $\boldsymbol{b}_s$ and $\boldsymbol{j}_s$ are obtained from the unperturbed solution as

$$\boldsymbol{b}_s = (\mu_p - \mu_u)\, \boldsymbol{h}_u \ \text{ in } \Omega_{c,p}\,, \qquad (5)$$

$$\boldsymbol{j}_s = (\sigma_p - \sigma_u)\, \boldsymbol{e}_u \ \text{ in } \Omega_{c,p}\,. \qquad (6)$$

These sources only act in the regions where a change of conductivity or permeability occurs. The considered boundary conditions neglect the distortions at a certain distance from $\Omega_{c,i}$. For convenience, the perturbation problem does not take account of the geometrical and material details of the initial unperturbed problem. For close positions of source and perturbing regions, a more accurate solution could be obtained via an iterative procedure to calculate successive perturbations in each region, not only from the initial source region to the perturbing one but also from the latter to the former. At the discrete level, the meshes of both unperturbed and perturbation problems can then be significantly simplified, each problem asking for mesh refinement of different regions.

The source $\mathbf{b}_s$ (5), determined from the known field $\boldsymbol{h}_u$, is itself also zero in $\Omega_{c,i}$. The source current density $\boldsymbol{j}_s$ (6) is to be obtained from the unperturbed electric field $\mathbf{e}_u$, with $\sigma_u \to \infty$ and σ_p finite in $\Omega_{c,i}$. The quantities involved in (6) are, on the one hand, $\sigma_u \mathbf{e}_u$ being the surface current density on $\partial\Omega_{c,i}$, and, on the other hand, $\sigma_p \mathbf{e}_u$ actually being null in $\Omega_{c,i}$ because $\mathbf{e}_u$ is only defined on $\partial\Omega_{c,i}$. Consequently, one has to consider $\boldsymbol{j}_s$ as a surface source current density, i.e. $\boldsymbol{j}_s = -\sigma_u\, \boldsymbol{e}_u$ on $\partial\Omega_{c,i}$.

B. The weak formulations

The eddy current problem is defined in Ω with the magnetic vector potential formulation [4], expressed in terms of a magnetic vector potential $\boldsymbol{a}$ and an electric scalar potential v, i.e.

$$(\mu^{-1}\,\mathrm{curl}\,\boldsymbol{a}, \mathrm{curl}\,\boldsymbol{a}')_{\Omega} + (\sigma\,\partial_t \boldsymbol{a}, \boldsymbol{a}')_{\Omega_c} + (\sigma\,\mathrm{grad}\,v, \boldsymbol{a}')_{\Omega_c}$$
$$+ < \boldsymbol{n} \times \boldsymbol{h}_s, \boldsymbol{a}' >_{\Gamma_h} = 0\,,\ \forall \boldsymbol{a}' \in F^1(\Omega)\,. \qquad (7)$$

The surface integral term on Γ_h in (7) accounts for the natural boundary or interface conditions. It will be shown to be of key importance in our developments.

The BC on the perfect conductors can be expressed via the definition of a surface scalar potential u_u for the primary unknown $\boldsymbol{a}_u$ [5]. In a 2D model, with perpendicular currents, this amounts to define a floating (constant) value for the perpendicular component of $\boldsymbol{a}_u$ for each conductor.

The unperturbed formulation is of the form (7) with all the quantities with the subscript u. The surface integral term on $\partial\Omega_{c,i}$ can be shown to be the total current I_i flowing on $\partial\Omega_{c,i}$ for the test function relative to u_u, which enables the coupling with electrical circuits.

The skin and proximity effects in each conductor $\Omega_{c,i}$ have then to be modified and adapted for their actual conducting nature. This is done via the weak formulation of the perturbation problem of a form similar to (7). Each of its surface integral terms related to $\partial\Omega_{c,i}$ is actually known from the unperturbed solution. It will be shown to be determined in a natural weak way from a volume integral coming from the unperturbed problem.

III. Application and Discussions

A core-inductor system is considered as a test problem, gathering both active and passive conductors. Holes in the core are considered in order to point out the effect of several corners.

Fig. 1 shows the eddy current distribution along the core surface for particular working conditions presenting a significant skin effect. Fig. 2 highlights the relative error on this current density and the associated Joule power density made by the impedance BC technique versus the sub-domain FE approach. The error significantly increases in the vicinity of the conductor corners: it exceeds 50% for the Joule power density and 30% for the current density in the smallest plane portions between holes. This affects the total losses accuracy when the size of the conductor portions decreases. The error with the impedance BC technique is shown to be significant up to a distance of about 3 δ_{Al} from each corner, whereas a good accuracy is only obtained beyond this distance. Other results will be presented and discussed regarding the current distributions in both the inductor and the core for different working conditions. The developed technique will be validated and its main advantages versus the impedance BC technique will be pointed out.

Once calculated, the source limit solution can be used in each subproblem not only for a single high frequency signal but for several signals. This allows efficient parameterized analyses on the signal form and the electric and magnetic characteristics of the conductors in a wide range, i.e. on all the parameters affecting the skin depth. Nonlinear analyses can also benefit from this.

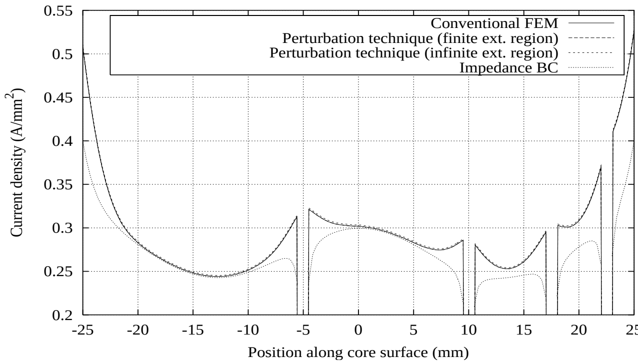

Fig. 1. Eddy current density along the core surface for the conventional FE solution, the perturbation technique and the impedance BC technique.

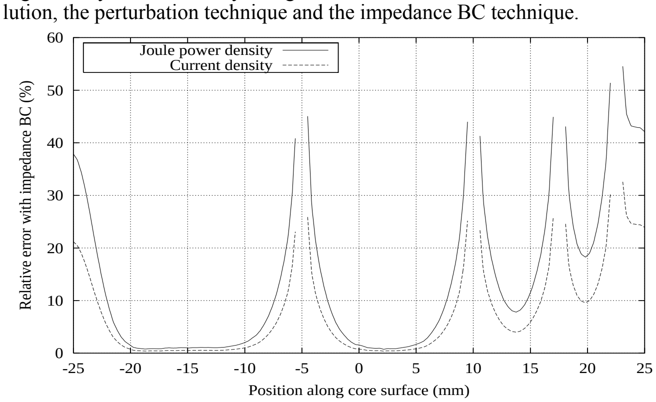

Fig. 2. Relative error on the current density and the associated Joule power density along the core surface made by the impedance BC technique versus the sub-domain FE approach.

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